LEBANESE AMERICAN UNIVERSITY
Department of Computer Science and Mathematics

Discrete Mathematical Structures I

Exam II **Fall 2013 (Dec 2013)**

Name: Solutions **ID:**

Question Number	**Grade**
1. 16%	
2. 17%	
3. 16%	
4. 9%	
5. 9%	
6. 9%	
7. 9%	
8. 15%	
Total	

16/.

1. Show by induction that $1.3 + 2.4 + 3.5 + \ldots n(n+2) = \frac{n(n+1)(2n+7)}{6}$

1) Basic step $n = 1$.

$1.(3) = \frac{1(2)(9)}{6}$ ✓

2) Assume True for $P(K)$. Show $P(K+1)$.

Assume $1.3 + 2.4 + \ldots + k(k+2) = \frac{K(K+1)(2K+7)}{6}$

show $\underbrace{1.3 + 2.4 + \ldots + K(K+2)}+ (K+1)(K+3) \overset{?}{=} \frac{(K+1)(K+2)(2K+9)}{6}$

$\frac{K(K+1)(2K+7)}{6} + (K+1)(K+3) \overset{?}{=} \frac{(K+1)(K+2)(2K+9)}{6}$

$K(K\cancel{+1})(2K+7) + 6(K+1)(K+3) \overset{?}{=} (K+1)(K+2)(2K+9)$

$(K+1)\left[K(2K+7) + 6(K+3)\right] \overset{?}{=} (K+1)(K+2)(2K+9)$. ✓

17/17.

2. Consider the sequence $\{a_n\}$ defined by: $a_0 = 1, a_1 = 2, a_2 = 3$, and $a_n = a_{n-2} + 2a_{n-3}$ for $n \geq 3$.

(a) Calculate a_n for $n = 3, 4$

$$a_3 = a_1 + 2a_0 = 2 + 2 = 4$$

$$a_4 = a_2 + 2a_1 = 3 + 4 = 7$$

(b) Prove that $a_n > \left(\frac{3}{2}\right)^n$, using induction for $n \geqslant 3$.

1) BASIC step $n=3$

$$a_3 = 4 \overset{?}{>} \left(\frac{3}{2}\right)^3 \qquad 4 \overset{?}{>} \frac{27}{8}$$

$$32 > 27 \checkmark$$

2) Ind. Step: Assume $a_i > \left(\frac{3}{2}\right)^i$, for $i = 1, 2, \ldots k$.

show $a_{k+1} > \left(\frac{3}{2}\right)^{k+1}$

$$a_{k+1} = a_{k-1} + 2a_{k-2}$$

$$> \left(\frac{3}{2}\right)^{k-1} + 2\left(\frac{3}{2}\right)^{k-2} = \left(\frac{3}{2}\right)^{k-2}\left[\frac{3}{2} + 2\right] = \left(\frac{3}{2}\right)^{k-2}\left[\frac{7}{2}\right]$$

But $\frac{7}{2} > \left(\frac{3}{2}\right)^3 = \frac{27}{8}$?, Yes since $56 > 54$ $\therefore$

$$a_{k+1} > \left(\frac{3}{2}\right)^{k-2}\left(\frac{3}{2}\right)^3 \Rightarrow a_{k+1} > \left(\frac{3}{2}\right)^{k+1} \checkmark$$

3. $f : N \times N \to N \times N$ be given by $f(m,n) = (4^m, m+n)$

(a) Is f 1-1?

Yes; if $f(m,n) = f(a,b) \Rightarrow$

$(4^m, m+n) = (4^a, a+b) \Rightarrow \quad a = m \Rightarrow n = b.$

$\Rightarrow (m,n) = (a,b) \Rightarrow f : 1-1$

16%.

(b) Is f onto? No; f is not onto since, for instance.

$(1, 3)$ has no pre-image $\Rightarrow$

there is no (a,b) such that $(4^a, a+b) = (1,3)$.

or $4^a = 1$ has no solution in $\underline{\underline{N}}$!!

4. Find your own example of a function $f : N \to N$ that is onto but not 1-1.

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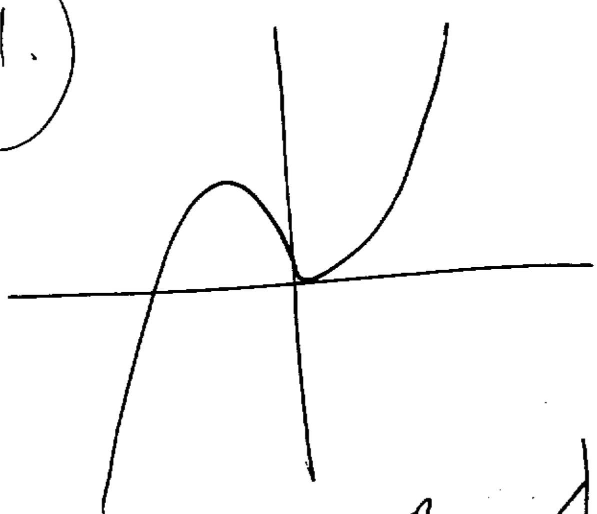
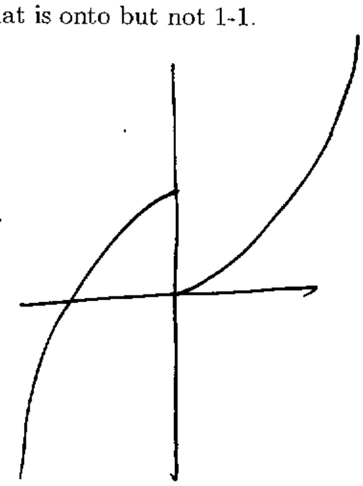


or



or

in this case $f(n) = \begin{cases} n+1 & \text{if } n<0 \\ n & \text{if } n \geq 0. \end{cases}$

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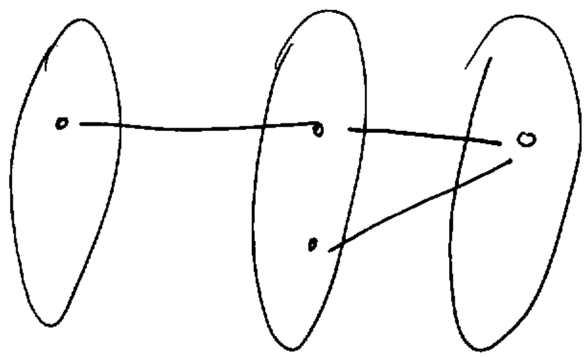
5. Let $f : A \to B$ and $g : B \to C$.and suppose that $gof : A \to C$ is onto. Show whether or not both functions f and g should also be onto.

No f need not be onto



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6. Find two 3x3 matrices A and B whose product is 0, yet non of them is the zero matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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7. If $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & -2 \\ 1 & 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 & 4 \\ 5 & -1 & 2 \\ 3 & 2 & -1 \end{bmatrix}$,evaluate: $2A + 3B^T$.

$$\begin{bmatrix} 2 & 6 & -2 \\ 0 & 4 & -4 \\ 2 & 2 & -2 \end{bmatrix} + 3\begin{bmatrix} 0 & 5 & 3 \\ 0 & -1 & 2 \\ 4 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 21 & 7 \\ 0 & 1 & 2 \\ 14 & 8 & -5 \end{bmatrix}$$

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8. Show that $\lceil x \rceil - \lfloor x \rfloor = 1$ when x is not an integer.

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$n < x < n+1$ $\quad x = n + d \quad 0 < d < 1$

$\lceil x \rceil = n+1 \quad \lfloor x \rfloor = n$

$\Rightarrow \lceil x \rceil - \lfloor x \rfloor = 1$ since

$(n+1) - (n) = 1$